PHYSICS

Motion in a Plane

Vectors

1. (2)

$|\vec{A}+\vec{B}|=|\vec{A}-\vec{B}|$

Squaring on both sides $|\vec{A}+\vec{B}|^2=|\vec{A}-\vec{B}|^2$

$\Rightarrow \vec{A}\cdot\vec{A}+2\vec{A}\cdot\vec{B}+\vec{B}\cdot\vec{B}$

$\Rightarrow \vec{A}\cdot\vec{A}-2\vec{A}\cdot\vec{B}+\vec{B}\cdot\vec{B}$

$\Rightarrow 4\vec{A}\cdot\vec{B}=0 \Rightarrow 4AB\cos\theta=0$

$\Rightarrow \cos\theta=0 \Rightarrow \theta=90°$

2. (4)

$\vec{v}_{av}=\dfrac{\Delta\vec{r}(\text{Displacement})}{\Delta t(\text{Time taken})}$

$=\dfrac{(13-2)\hat{i}+(14-3)\hat{j}}{5-0}=\dfrac{11}{5}(\hat{i}+\hat{j})$

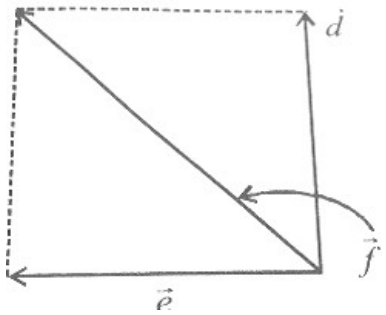
When a point have coordinate (x, y) then its position vector = $x\hat{i}+y\hat{j}$

When a particle moves from point (x_1, y_1) to (x_2, y_2), then its displacement vector

$\vec{r}=(x_2-x_1)\hat{i}+(y_2-y_1)\hat{j}$

3. (3)

Using the law of vector addition, $(\vec{d}+\vec{e})$ is as shown in the figure.



$\therefore \vec{d}+\vec{e}=\vec{f}$

4. (4)

$|\vec{A}\times\vec{B}|=-=\sqrt{3}(\vec{A}\cdot\vec{B})$

$\Rightarrow AB\sin\theta=\sqrt{3}AB\cos\theta$

$\Rightarrow \tan\theta=\sqrt{3} \Rightarrow \theta=60°$

5. (4)

$|\vec{A}+\vec{B}|^2=|\vec{A}-\vec{B}|^{-2}$

$=|\vec{A}|^2+|\vec{B}|^2+2\vec{A}\cdot\vec{B}$

$=A^2+B^2+2AB\cos\theta$

$=|\vec{A}-\vec{B}|^2=|\vec{A}|^2+|\vec{B}|^2-2\vec{A}\cdot\vec{B}$

$=A^2+B^2-2AB\cos\theta$

So, $A^2+B^2+2AB\cos\theta$

$=A^2+B^2-2AB\cos\theta$

$4AB\cos\theta=0 \Rightarrow \cos\theta=0 \Rightarrow \theta=90°$

So, angle between A and B is 90°.

6. (2)

For two vectors to be perpendicular to each other. $\vec{A}\cdot\vec{B}=0$

$(2\hat{i}+3\hat{j}+8\hat{k})\cdot(4\hat{j}-4\hat{i}+\alpha\hat{k})=0$

$-8+12+8\alpha=0$

$\alpha=-\dfrac{4}{8}=-\dfrac{1}{2}$

7. (3)

$\vec{P}=$ vector sum $=\vec{A}+\vec{B}$

$\vec{Q}=$ Vector differences $=\vec{A}-\vec{B}$

Since, $\vec{P}$ and $\vec{Q}$ are perpendicular

$\therefore \vec{P}\cdot\vec{Q}=0 \Rightarrow (\vec{A}+\vec{B})\cdot(\vec{A}-\vec{B})=0$

$\Rightarrow A^2=B^2 \Rightarrow |\vec{A}|=|\vec{B}|$

8. (3)

$\vec{A}=3\hat{i}+4\hat{j}+5\hat{k}$ and $\vec{B}=3\hat{i}+4\hat{j}-5\hat{k}$

$\vec{A}\cdot\vec{B}=(3\hat{i}+4\hat{j}+5\hat{k})\cdot(3\hat{i}+4\hat{j}-5\hat{k})$

$|\vec{A}|\cdot|\vec{B}|\cos\theta=9+16-25=0$

$|\vec{A}|\neq 0, |\vec{B}|\neq 0$

Hence, $\cos\theta=0$, $\theta=90°$

9. (2)

$\hat{r}=0.5\hat{i}+0.8\hat{j}+c\hat{k}$

$|\hat{r}|=1=\sqrt{(0.5)^2+(0.8)^2+c^2}$

$(0.5)^2+(0.8)^2+c^2=1$

$c^2=0.11 \Rightarrow c=\sqrt{0.11}$

Unit vector is a vector which has a magnitude of one. It is a vector divided by its magnitude. Unit vector for

$\vec{A}$ is $\hat{A}:\hat{A}=\dfrac{\vec{A}}{A}$

Unit vector gives direction.

10. (3)